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Office 519

Number theory

Fall Semester 2013
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FINAL EXAM (1h50)

Maximal Score: 200 points

Show ALL steps and make sure I understand how you get the answer to have full credit! No material allowed!

Wednesday december 18th

Problem 1: ($\star$) 10 points
Let $a = 2^4 13^2 19$ and $b = 2^3 5^2 13$. Give the prime factorization of $gcd(a^2, b^3)$

Problem 2: ($\star$) 10 points
Show that $4(29!) + 5!$ is divisible by 31. (Hint: Wilson's theorem!)

Problem 3: ($\star$) 10 points
Give a non-trivial factor of $2^{52} + 1$.

Problem 4: ($\star$) 10 points
Find all right angled triangles with coprime integer sides and base of length 28. (Hint: Pythagorean triples!)

Problem 5: ($\star$) 10 points
Prove that $\sqrt{7}$ is irrational. (Hint: By contradiction!)

Problem 6: ($\star$) 10 points Suppose that $n^2 = \sum_{d|n} f(d)$. Evaluate $f(8)$.

Problem 7: ($\star$) 15 points You have chosen to do RSA cryptography with modulus $n = pq$ where $p = 7$ and $q = 19$

1. Compute the least common multiple $[\phi(p), \phi(q)]$.
2. Suppose that the encode exponent is $e = 5$. Calculate a decode exponent d.

Problem 8: ($\star$) 20 points

1. Use the Eulidean algorithm to compute the greatest common divisor $(263, 271)$.
2. Solve the linear equation $263x - 271y = 5$ or explain why there are no solutions.

Problem 9: ($\star$) 10 points Solve the simultaneous congruences equations :

$$\begin{cases} x \equiv 5 \ mod\ 7 \\ x \equiv 2 \ mod\ 5 \end{cases}$$

Problem 10: ($\star$) 20 points Say if the following Gaussian integers are prime and when they are not give a prime factorization in $\mathbb{Z}[i]$:

1. 7
2. $1+2i$

Problem 11: ($\star$) 15 points

1. Evaluate $\phi(1500)$.
2. Compute the remainder when 7^{1203} is divided by 1500.

Problem 12: ($\star$) 10 points Use induction to prime that $6^n \equiv 5n+1 \ mod\ 25$ for all positive integer n.

Problem 13: ($\star$) 20 points

1. Find the continued fraction expansion of $\sqrt{30}$.
2. Find the quadratic α with continued fraction expansion $\alpha = [\overline{2,3}]$.

Problem 14: ($\star$) 30 points

1. Evaluate $\left(\frac{293}{331}\right)$.
2. Prove that the Diophantine equation $x^2+y^2 = 12z+7$ does not have any solution. (Hint: modulo 12.)
3. Prove that the quadratic congruence $x^2 - 4xy + 5y^2 \equiv 0 \ mod\ 11$ has no solution.

[1]

[1]($\star$) = easy , ($\star\star$)= medium, ($\star\star\star$)= challenge